

# Implications of Minimal Length Scale on the Statistical Mechanics of Ideal Gas

Kourosh Nozari and S. Hamid Mehdipour

*Department of Physics, Faculty of Basic Science,  
University of Mazandaran,  
P. O. Box 47416-1467, Babolsar, IRAN  
e-mail: knozari@umz.ac.ir*

## Abstract

Several alternative approaches to quantum gravity problem suggest the modification of the *fundamental volume*  $\omega_0$  of the accessible phase space for representative points. This modified fundamental volume has a novel momentum dependence. In this paper, we study the effects of this modification on the thermodynamics of an ideal gas within the microcanonical ensemble and using the generalized uncertainty principle(GUP). Although the induced modifications are important only in quantum gravity era, possible experimental manifestation of these effects may provides strong support for underlying quantum gravity proposal.

**PACS:** 04.60.-m, 05.70.Ce, 51.30.+i

**Key Words:** Quantum Gravity, Generalized Uncertainty Principle, Statistical Mechanics, Ideal Gas

# 1 Motivation

In ordinary statistical mechanics, it is impossible to define the position of a representative point in the phase space of the given system more accurately than the situation which is given by  $(\delta q \delta p)_{min} \geq \hbar$ . In another words, around any point  $(q, p)$  of the (two dimensional) phase space, there exists an area of the order  $\hbar$  which the position of the representative point cannot be pin-pointed. In a phase space of  $2N$  dimensions, the corresponding volume of uncertainty around any point would be of order  $\hbar^N$ . Therefore, it is reasonable to regard the phase space as made up of elementary cells of volume  $\approx \hbar^N$ . These cells have one-to-one correspondence with the quantum mechanical states of the given system[1].

In ordinary picture of quantum theory, we usually ignore the gravitational effects. Nevertheless, gravity induces uncertainty. Combining this extra uncertainty with usual uncertainty principle of Heisenberg, we find the generalized uncertainty principle(GUP). Therefore, measurements in quantum gravity should be governed by GUP. Much evidence exists (from string theory[2-6], non-commutative geometry[7], loop quantum gravity[8] and black hole physics[9]) which confirm GUP. On the other hand, theories such as scale relativity of Nottale[10], Twistors theory of Penrose[11] and  $E$ -infinity of El Naschie[12-14] provide strong supports and deep understanding of this finite resolution of spacetime points. All of this evidence has its origin in the quantum fluctuations of the background spacetime metric(the so-called spacetime fuzziness[15,16]-spacetime fuzziness with given mathematical exactness in fuzzy  $K3$  manifold of  $\mathcal{E}^\infty$ [17]- and/or foamy/fractal spacetime [18]). A common feature of all promising candidates for quantum gravity is the existence of a minimal length scale on the order of Planck length. This intriguing aspect of quantum gravity has been investigated from different perspectives[19-31]. A generalized uncertainty principle can be formulated as

$$\delta x \delta p \geq \hbar \left(1 + \beta (\delta p)^2\right). \quad (1)$$

(Note that actually we should consider an extra term on the right hand side of this equation which contains expectation value of  $p$ . Since we are dealing with absolute value of non-vanishing minimal uncertainty in position, we have set this term to be zero. For complete discussion of this point see [15]). The main result of this GUP is the existence of a non-vanishing minimal observable length which is a consequence of finite resolution of spacetime at Planck scale. In fact, position measurement is possible only up to a multiple

of Planck length,  $l_p = \sqrt{\frac{G\hbar}{c^3}} \sim 10^{-33}cm$ [15]. A simple calculation based on (1) shows that

$$(\Delta x)_{min} = \hbar\sqrt{\beta}.$$

Therefore  $\beta$  is related to minimal observable length. In the spirit of string theory this minimal length is of the order of string length. It is impossible to set up a measurement to find more accurate particle position than Planck length, and this means that the notion of locality breaks down(the so-called spacetime fuzziness)(see[32] and references therein). For our purposes, this generalized uncertainty principle can be rewritten as follows

$$\delta q \delta p \geq \hbar(1 + \beta(p)^2). \quad (2)$$

Note that one can interpret this result as a generalization of  $\hbar$  (this generalization can lead to generalization of De Broglie principle[33]). Now it is obvious that the volume of uncertainty around any point of phase space increases due to extra term in the right hand side of (2)( or equivalently due to generalization of  $\hbar$ ). This generalized volume is given by  $[\hbar(1 + \beta p^2)]^N$ . Since  $\beta$  is a positive quantity, this feature will decrease drastically the number of accessible microstates for a given system specially in high momentum limit. Our goal here is to calculate this reduction within microcanonical ensemble and investigate its consequences. We consider a simple system of monatomic ideal gas in the case of microcanonical ensemble. This issue firstly has been considered by Kalyana Rama[34]. He has discussed the effect of GUP on various thermodynamical quantities in *grand canonical ensemble*. Here we are going to consider the effects of GUP on thermodynamics of ideal gas in *microcanonical ensemble*. Note that an elegant formulation of statistical mechanics of multi-dimensional Cantor sets based on fractal nature of spacetime has been provided by El Naschie[35].

In which follows we consider GUP as our primary input. Although GUP is a model-independent concept, its functional form is quantum gravity model dependent. As a result, there are more generalization of GUP which consider further terms on the right hand side of equation (1)(see [15]), but in some sense regarding dynamics, equation (1) has more powerful physical grounds(from string theoretical view point, see[6]). Note that GUP (1) has a non-vanishing minimal uncertainty only in position; there is no non-vanishing minimal uncertainty in momentum. Since aforementioned reduction of accessible phase space is a quantum gravity effect, any experimental test of quantum gravity signals will support our findings.

## 2 GUP and Thermodynamics of Ideal Gas

To illustrate the concepts developed in the preceding section, we derive here the thermodynamical properties of an ideal gas composed of monatomic noninteracting particles within GUP. In microcanonical ensemble, the macrostate of the given system is defined by the number of molecules  $N$ , the volume  $V$  and energy  $E$  of the system. In this ensemble, the volume  $\omega$  of the phase space accessible to the representative points of the (member) system where have a choice to lie anywhere within a *hypershell* defined by the condition  $E - \frac{\Delta}{2} \leq H(q, p) \leq E + \frac{\Delta}{2}$  is given by

$$\omega = \int' d\omega = \int' \int' (d^{3N} q) (d^{3N} p) \quad (3)$$

where  $\omega \equiv \omega(N, V, E; \Delta)$ , and the primed integration extends only over that part of the phase space which conforms to the above condition. Since the Hamiltonian in this case is a function of the  $p$ 's alone, the integrations over the  $q$ 's can be carried out straightforwardly which gives a factor of  $V^N$ . The remaining integral

$$\int' (d^{3N} p) \quad (4)$$

should be evaluated under the following condition

$$2m \left[ E - \frac{\Delta}{2} \right] \leq \sum_{i=1}^{3N} p_i^2 \leq 2m \left[ E + \frac{\Delta}{2} \right]. \quad (5)$$

Now we should consider the following two key points[32,33],[36-38]:

- Within GUP framework, particle's momentum generalizes. This generalized momentum is given by

$$p^{GUP} \simeq p \left( 1 + \frac{1}{3} \beta p^2 \right) \quad (6)$$

- Due to generalization of momentum, energy will generalize too

$$E^{GUP} \simeq E \left( 1 + \frac{1}{3} \beta E^2 \right). \quad (7)$$

Note that for simplicity we have considered only first order corrections. Higher order corrections lead to integrals which can be calculated only with sophisticated numerical scheme. With this two points in mind, up to first order in  $\beta$ , the *hypershell* equation is given by

$$2m \left[ E \left( 1 + \frac{1}{3} \beta E^2 \right) - \frac{\Delta}{2} \right] \leq \sum_{i=1}^{3N} p_i^2 \left( 1 + \frac{2}{3} \beta p_i^2 \right) \leq 2m \left[ E \left( 1 + \frac{1}{3} \beta E^2 \right) + \frac{\Delta}{2} \right]. \quad (8)$$

Now, integral (4) is equal to the volume of a  $3N$ -dimensional hypershell, bounded by two hyperspheres of radii

$$\sqrt{2m\left[E(1 + \frac{1}{3}\beta E^2) - \frac{\Delta}{2}\right]} \quad \text{and} \quad \sqrt{2m\left[E(1 + \frac{1}{3}\beta E^2) + \frac{\Delta}{2}\right]}.$$

Therefore, we can write

$$\int' \dots \int' \prod_{i=1}^{3N} dp_i = C_{3N} \left( \sqrt{2m(E^{GUP} + \frac{\Delta}{2})} \right)^{3N} := \Lambda \quad (9)$$

where

$$0 \leq \sum_{i=1}^{3N} p_i^2 (1 + \frac{2}{3}\beta p_i^2) \leq 2m(E^{GUP} + \frac{\Delta}{2}).$$

This statement gives half of the volume of relevant phase space (since  $\Delta \ll E^{GUP}$ ) and therefore we should multiply our final result with a factor of 2. Here  $C_{3N}$  is a constant which depends only on the dimensionality of phase space. Clearly, the volume element  $d\Lambda$  can also be written as

$$d\Lambda = \frac{3}{2} N C_{3N} (\sqrt{2m})^{3N} \left[ \sqrt{E^{GUP} + \frac{\Delta}{2}} \right]^{3N-2} dE^{GUP}. \quad (10)$$

To evaluate  $C_{3N}$ , we use the following integral formula,

$$\int_{-\infty}^{+\infty} \exp(-p^2 - \frac{2}{3}\beta p^4) dp = \frac{1}{2} \left( \frac{3}{2\beta} \right)^{\frac{1}{2}} \exp\left(\frac{3}{16\beta}\right) K_{\frac{1}{4}}\left(\frac{3}{16\beta}\right), \quad (11)$$

where  $K_\nu(x)$  is the modified Bessel function of the second kind. Multiplying  $3N$  such integrals, one for each of variables  $p_i$ , we obtain

$$\int_{-\infty}^{+\infty} \dots \int_{-\infty}^{\infty} \exp\left(-\sum_{i=1}^{3N} p_i^2 (1 + \frac{2}{3}\beta p_i^2)\right) \prod_{i=1}^{3N} dp_i = \left[ \frac{1}{2} \left( \frac{3}{2\beta} \right)^{\frac{1}{2}} \exp\left(\frac{3}{16\beta}\right) K_{\frac{1}{4}}\left(\frac{3}{16\beta}\right) \right]^{3N}. \quad (12)$$

Therefore, it follows that

$$\int_{-\infty}^{+\infty} \exp\left(-2m\left(E^{GUP} + \frac{\Delta}{2}\right)\right) d\Lambda = \left[ \frac{1}{2} \left( \frac{3}{2\beta} \right)^{\frac{1}{2}} \exp\left(\frac{3}{16\beta}\right) K_{\frac{1}{4}}\left(\frac{3}{16\beta}\right) \right]^{3N}. \quad (13)$$

Using equation (10), we find

$$\int_{-\infty}^{+\infty} \frac{3}{2} N C_{3N} (\sqrt{2m})^{3N} \left[ \sqrt{E^{GUP} + \frac{\Delta}{2}} \right]^{3N-2} \exp\left(-2m\left(E^{GUP} + \frac{\Delta}{2}\right)\right) dE^{GUP} = [n(\beta)]^{3N}, \quad (14)$$

where for simplicity we have defined  $n(\beta) = \frac{1}{2} \left( \frac{3}{2\beta} \right)^{\frac{1}{2}} \exp \left( \frac{3}{16\beta} \right) K_{\frac{1}{4}} \left( \frac{3}{16\beta} \right)$ . Calculation of this integral gives

$$C_{3N} = \frac{2[n(\beta)]^{3N} \exp(m\Delta)}{3N(2m)^{\frac{3N}{2}} \int_{-\infty}^{+\infty} (E^{GUP} + \frac{\Delta}{2})^{\frac{3N-2}{2}} \exp(-2mE^{GUP}) dE^{GUP}}. \quad (15)$$

For  $\Delta \ll E^{GUP}$ , this equation reduces to

$$C_{3N} = \frac{2[n(\beta)]^{3N}}{3N(\frac{3N}{2} - 1)!}. \quad (16)$$

Now, from equation (9), we find (since  $N \gg 1$ )

$$\int \dots \int \prod_{i=1}^{3N} dp_i \equiv \frac{2[n(\beta)]^{3N} (2mE^{GUP})^{\frac{3N}{2}} \left[ 1 + \frac{3N\Delta}{4E^{GUP}} \right]}{3N(\frac{3N}{2} - 1)!} \simeq \frac{\Delta}{2E^{GUP}} \frac{[n(\beta)]^{3N}}{(\frac{3N}{2} - 1)!} (2mE^{GUP})^{\frac{3N}{2}}. \quad (17)$$

Hence, the total volume of the phase space enclosed within hypershell is given by

$$\omega \simeq \frac{\Delta}{E^{GUP}} V^N \frac{(2[n(\beta)]^2 m E^{GUP})^{\frac{3N}{2}}}{(\frac{3N}{2} - 1)!}. \quad (18)$$

There exists a direct correspondence between the various microstates of the given system and the various locations in the phase space. The volume  $\omega$  (of the allowed region of the phase space) is, therefore, a direct measure of the multiplicity  $\Omega$  of the microstates obtaining in the system. To establish a numerical correspondence between  $\omega$  and  $\Omega$ , we should discover a *fundamental volume*  $\omega_0$  which could be regarded as equivalent to one microstate. Once this is done, we can right away conclude that, asymptotically,

$$\Omega = \frac{\omega}{\omega_0}, \quad (19)$$

where in GUP,  $\omega_0$  is given by

$$\omega_0 = (\delta q \delta p)^{3N} = [\hbar(1 + \beta p^2)]^{3N} \equiv \hbar'^{3N}, \quad (20)$$

Finally, we find the multiplicity (total number of microstates) of the given system as

$$\Omega = \frac{V^N}{\hbar'^{3N}} \frac{\Delta}{E^{GUP}} \frac{(2[n(\beta)]^2 m E^{GUP})^{\frac{3N}{2}}}{(\frac{3N}{2} - 1)!}. \quad (21)$$

Apparently, within GUP, due to increased fundamental volume  $\omega_0$ , number of total microstates decreases. This point have been discussed in more powerful manner in terms of

information loss on the quantum scale of spacetime by El Naschie[35]. In fact based on El Naschie point of view, there is a fuzzy region information related to the de Broglie length followed by an informational horizon given by the loss of information. In this manner he find a bound for this information loss(see reference [35] page 207).

The complete thermodynamics of the given system would then follow in the usual way, namely through the relationship,

$$S(N, V, E^{GUP}) = k \ln \Omega = k \ln \left( \frac{V^N}{\hbar^{3N}} \frac{\Delta}{E^{GUP}} \frac{(2[n(\beta)]^2 m E^{GUP})^{\frac{3N}{2}}}{(\frac{3N}{2} - 1)!} \right). \quad (22)$$

In the absence of quantum gravitational effects, that is when  $\beta = 0$ , we obtain standard thermodynamical results(note that  $n(\beta)$  tends to  $\sqrt{\pi}$  when  $\beta \rightarrow 0$ ). Various thermodynamical quantities can then be calculated using equation(22) straightforwardly.

Our analysis shows a reduction of the number of accessible microstates in high momentum regime. This reduction of has novel implications such as reduction of corresponding microcanonical entropy of the system. It seems that thermodynamical systems at very short distances have an "unusual thermodynamics". Recently, we have shown[39], for a micro-black hole which lies in the spirit of such very small scale systems, that the temperature has an unusual behavior and entropy tends to zero when the size of the system tends to Planck length. Here, it is evident that in high momentum regime the entropy of ideal gas tends to zero which resembles the mentioned behavior of micro-black hole. This seems to be a universal behavior of short distance physics.

Note that our analysis is based on the generalization of momentum  $p$  and energy  $E$ (see equations (6) and (7)). These generalizations lead to modified dispersion relations. Modification of dispersion relations is motivated by several quantum gravity scenarios(see [32] and [36-38]). Searches for modifications of the dispersion relation of the form (6) or (7) (and their more general forms), constitute part of experimental quantum gravity efforts. This type of effect is beyond the standard model extension, since it could not be described by power counting renormalizable terms. A possible manifestation of the modified dispersion relations is an energy dependent propagation velocity which should lead to different time-of-arrivals of the same event on a distant star when looked at it in different frequency channels[40](also see [32] and references therein). Another effect is frequency dependent position of interference fringes in interferometric experiments. These are some possible experimental schemes for detecting such quantum gravity signals. Although these experimental scheme are not direct consequence of our findings, but their

possible support of modified dispersion relations can be considered as an indirect support of our finding. Therefore it is possible in principle to test reduction of phase space due to quantum gravity effects.

### 3 GUP and Thermodynamics of Extreme Relativistic Gas

In this section, we calculate thermodynamics of an ultra-relativistic monatomic noninteracting gaseous system within GUP and up to first order corrections. Using arguments presented in preceding section, the hypershell equation for ultra-relativistic gaseous system is given by

$$\frac{1}{c} \left[ E(1 + \frac{1}{3}\beta E^2) - \frac{\Delta}{2} \right] \leq \sum_{i=1}^{3N} p_i(1 + \frac{1}{3}\beta p_i^2) \leq \frac{1}{c} \left[ E(1 + \frac{1}{3}\beta E^2) + \frac{\Delta}{2} \right]. \quad (23)$$

Now integral (4) is equal to the volume of a  $3N$ -dimensional hypershell, bounded by two hyperspheres of radii

$$\sqrt{\frac{1}{c} \left[ E(1 + \frac{1}{3}\beta E^2) - \frac{\Delta}{2} \right]} \quad \text{and} \quad \sqrt{\frac{1}{c} \left[ E(1 + \frac{1}{3}\beta E^2) + \frac{\Delta}{2} \right]}.$$

The number of accessible microstates for the system is proportional to the volume of this hypershell. In the same manner as previous section, we have

$$\int' \dots \int' \prod_{i=1}^{3N} dp_i = D_{3N} \left( \sqrt{\frac{1}{c} (E^{GUP} + \frac{\Delta}{2})} \right)^{3N} := \Upsilon, \quad (24)$$

where

$$0 \leq \sum_{i=1}^{3N} p_i(1 + \frac{1}{3}\beta p_i^2) \leq \frac{1}{c} (E^{GUP} + \frac{\Delta}{2}).$$

Here  $D_{3N}$ , is a constant which depends only to the dimensionality of the phase space. Now, since  $d\Upsilon$  can be written as

$$d\Upsilon = \frac{3}{2} N D_{3N} \left( \frac{1}{c} \right)^{\frac{3N}{2}} \left[ \sqrt{E^{GUP} + \frac{\Delta}{2}} \right]^{3N-2} dE^{GUP}, \quad (25)$$

to evaluate  $D_{3N}$ , we use the following integral formula

$$\int_{-\infty}^{+\infty} \exp(-p - \frac{1}{3}\beta p^3) dp = m(\beta), \quad (26)$$

where,  $m(\beta) = \frac{2}{3\sqrt{\beta}} \left[ \mathcal{LS}2(0, \frac{1}{3}, \frac{2}{3\sqrt{\beta}}) + \mathcal{LS}2(0, \frac{1}{3}, -\frac{2}{3\sqrt{\beta}}) \right]$  is a linear combination of Lommel functions. Note that we should consider the real part of this combination. Multiplying  $3N$  such integrals, one for each of variables  $p_i$ , we obtain

$$\int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} \exp \left( - \sum_{i=1}^{3N} p_i \left( 1 + \frac{1}{3} \beta p_i^2 \right) \right) \prod_{i=1}^{3N} dp_i = [m(\beta)]^{3N}, \quad (27)$$

whence it follows that,

$$\int_{-\infty}^{+\infty} \exp \left( - \frac{1}{c} \left( E^{GUP} + \frac{\Delta}{2} \right) \right) dR_{3N}(R) = [m(\beta)]^{3N}. \quad (28)$$

Now, using equation(25), we find

$$\int_{-\infty}^{+\infty} \frac{3}{2} N D_{3N} \left( \frac{1}{c} \right)^{\frac{3N}{2}} \left[ \sqrt{E^{GUP} + \frac{\Delta}{2}} \right]^{3N-2} \exp \left( - \frac{1}{c} \left( E^{GUP} + \frac{\Delta}{2} \right) \right) dE^{GUP} = [m(\beta)]^{3N}. \quad (29)$$

Therefore  $D_{3N}$  is given by

$$D_{3N} = \frac{2[m(\beta)]^{3N} \exp \left( \frac{\Delta}{2c} \right)}{3N \left( \frac{1}{c} \right)^{\frac{3N}{2}} \int_{-\infty}^{+\infty} \left( E^{GUP} + \frac{\Delta}{2} \right)^{\frac{3N-2}{2}} \exp \left( - \frac{1}{c} E^{GUP} \right) dE^{GUP}}. \quad (30)$$

For  $\Delta \ll E^{GUP}$  one obtains

$$D_{3N} = \frac{2[m(\beta)]^{3N}}{3N \left( \frac{3N}{2} - 1 \right)!}. \quad (31)$$

Now from equation (24) we have

$$\int \dots \int \prod_{i=1}^{3N} dp_i \equiv \frac{2[m(\beta)]^{3N} \left( \frac{1}{c} E^{GUP} \right)^{\frac{3N}{2}} \left[ 1 + \frac{3N\Delta}{4E^{GUP}} \right]}{3N \left( \frac{3N}{2} - 1 \right)!} \simeq \frac{\Delta}{2E^{GUP}} \frac{[m(\beta)]^{3N}}{\left( \frac{3N}{2} - 1 \right)!} \left( \frac{1}{c} E^{GUP} \right)^{\frac{3N}{2}}. \quad (32)$$

Hence, the total volume of the phase space enclosed within hypershell is given by

$$\omega \simeq \frac{\Delta}{E^{GUP}} V^N \frac{\left( \frac{1}{c} [m(\beta)]^2 E^{GUP} \right)^{\frac{3N}{2}}}{\left( \frac{3N}{2} - 1 \right)!}. \quad (33)$$

Since

$$\Omega = \frac{\omega}{\omega_0}, \quad (34)$$

we find finally

$$\Omega = \frac{V^N}{\hbar^{3N}} \frac{\Delta}{E^{GUP}} \frac{\left( \frac{1}{c} [m(\beta)]^2 E^{GUP} \right)^{\frac{3N}{2}}}{\left( \frac{3N}{2} - 1 \right)!}. \quad (35)$$

The thermodynamics of the system would then follow in the usual way through the following relation,

$$S(N, V, E^{GUP}) = k \ln \Omega = k \ln \left( \frac{V^N}{\hbar^{3N}} \frac{\Delta}{E^{GUP}} \frac{\left( \frac{1}{c} [m(\beta)]^2 E^{GUP} \right)^{\frac{3N}{2}}}{\left( \frac{3N}{2} - 1 \right)!} \right). \quad (36)$$

In the standard situation where  $\beta = 0$ , we obtain well-known results of ordinary statistical mechanics. Various thermodynamical quantities can then be calculated using equation(36). Once again, number of total microstates decreases leading to less entropy for given system in GUP.

## 4 Summary and Discussion

Generalized uncertainty principle(GUP) and/or modified dispersion relations(MDRs) are common features of all promising quantum gravity scenarios. As a result of GUP and/or MDRs, the fundamental volume  $\omega_0$  of accessible phase space for representative points of a given statistical system increases due to gravitational uncertainty. This increasing fundamental volume of accessible phase space can be interpreted as a result of Planck constant generalization. This feature requires a reformulation of statistical mechanics since number of accessible microstates for given system decreases drastically in quantum gravity era. This reduction of microstates is quantum gravity effect and can be avoided in standard situation but it plays important role in thermodynamics of early universe. Here we have considered an ideal gaseous system composed of noninteracting monatomic molecules within microcanonical ensemble. We have shown that GUP as a manifestation of quantum nature of gravity, leads to reduction of accessible microstates of the given system. We have considered two limits of classical and ultra-relativistic gas and in each case we have computed complete thermodynamics of the system.

Our analysis shows a reduction of the number of accessible microstates in high momentum regime. This reduction has novel implications such as reduction of corresponding microcanonical entropy of the system. In the limit of very high momentum, the entropy of the system tends to zero. It seems that thermodynamical systems at very short distances(very high energy) have an "unusual thermodynamics".

One can ask about the possible detection of these extra-ordinary effects. Up to now, there is no direct experimental or observational scheme for detection of these novel effects. Nevertheless, since the basis of our calculations come back to GUP and/or MDRs, possible

experimental schemes for detecting these quantum gravity signals are indirect tests of our findings. Several strategies for testing these quantum gravity predictions (GUP and/or MDRs) have been proposed [38]. Strategies such as frequency dependent position of interference fringes in interferometry experiments, an energy dependent propagation velocity which should lead to different time-of-arrivals of the same event on a distant star when looked at it in different frequency channels, possible detection of black hole remnants in Large Hadronic Collider (LHC) and also in ultrahigh energy cosmic ray (UHECR) air showers [41,42], are possible evidences for testing these novel quantum gravity effects. Therefore, any search for quantum gravity signals provides possible indirect test of generalized statistical mechanics which we have constructed.

## References

- [1] R. K. Pathria, *Statistical Mechanics*, Pergamon Press, First Edition, 1972
- [2] G. Veneziano, A stringy nature needs just two constants, *Europhys. Lett.* **2** (1986) 199; Proc. of Texas Superstring Workshop (1989).
- [3] D. Amati, M. Ciafaloni and G. Veneziano, Can spacetime be probed below the string size?, *Phys. Lett. B* **216** (1989) 41
- [4] D. Amati, M. Ciafaloni and G. Veneziano, Superstring collisions at Planckian energies, *Phys. Lett. B* **197** (1987) 81; Classical and quantum gravity effects from Planckian energy superstring collisions, *Int. J. Mod. Phys. A* **7** (1988) 1615; Higher-order gravitational deflection and soft bremsstrahlung in Planckian energy superstring collisions, *Nucl. Phys. B* **347** (1990) 530.
- [5] D. J. Gross and P. F. Mende, String theory beyond the Planck scale, *Nucl. Phys. B* **303** (1988) 407
- [6] K. Konishi, G. Paffuti and P. Provero, Minimum physical length and the generalized uncertainty principle in string theory, *Phys. Lett. B* **234** (1990) 276
- [7] S. Capozziello, G. Lambiase, G. Scarpetta, Generalized Uncertainty Principle from Quantum Geometry, *Int. J. Theor. Phys.* **39** (2000) 15.
- [8] L. J. Garay, Quantum gravity and Minimum length, *Int. J. Mod. Phys. A* **10** (1995) 145

- [9] F. Scardigli, Generalized Uncertainty Principle in Quantum Gravity from Micro-Black Hole Gedanken Experiment, *Phys. Lett. B***452** (1999) 39-44
- [10] L. Nottale, *Fractal space-time and microphysics: towards a theory of scale relativity*, World Scientific, Singapore, 1993
- [11] R. Penrose, The Central Programme of Twistor Theory, *Chaos, Solitons and Fractals*, **10** (1999) 581-611
- [12] M. S. El Naschie, The concepts of E-infinity: An elementary introduction to the Cantorian-fractal theory of quantum physics, *Chaos, Solitons and Fractals* **22** (2004) 495-511
- [13] M. S. El Naschie, Elementary prerequisites for E-infinity(Recommended background readings in nonlinear dynamics, geometry and topology), *Chaos, Solitons and Fractals*, **30** (2006) 579-605
- [14] - M. S. El Naschie, Intermediate prerequisites for E-infinity(Further recommended reading in nonlinear dynamics and mathematical physics), *Chaos, Solitons and Fractals*, **30** (2006) 622-628  
 - M. S. El Naschie, Advanced prerequisites for E-infinity theory, *Chaos, Solitons and Fractals*, **30** (2006) 636-641
- [15] A. Kempf, *et al.*, Hilbert space representation of the minimal length uncertainty relation, *Phys. Rev. D***52** (1995) 1108.
- [16] K. Nozari and B. Fazlpour, Some consequences of spacetime fuzziness, *Chaos, Solitons and Fractals*,(2006) (in Press)
- [17] M. S. El Naschie, Fuzzy Dodecahedron topology and E-infinity spacetime as a model for quantum physics, *Chaos, Solitons and Fractals*, **30** (2006) 1025-1033; Topics in the mathematical physics of E-infinity theory, *Chaos, Solitons and Fractals*, **30** (2006) 656-663; On two new fuzzy Kähler manifolds, Klein modular space and t' Hooft holographic principles, *Chaos, Solitons and Fractals*, **29** (2006) 876-881
- [18] G. Amelino-Camelia, A phenomenological description of quantum-gravity-induced space-time noise, *Nature* **410** (2001) 1065-1069

- [19] M. Maggiore, A Generalized Uncertainty Principle in Quantum Gravity , *Phys. Lett. B***304** (1993) 65.
- [20] M. Maggiore, Quantum Groups, Gravity, and the Generalized Uncertainty Principle, *Phys. Rev. D***49** (1994) 5182.
- [21] F. Scardigli, R. Casadio, Generalized Uncertainty Principle, Extra-dimensions and Holography, *Class. Quant. Grav.***20** (2003) 3915
- [22] S. Hossenfelder, Running Coupling with Minimal Length, *Phys. Rev. D***70** (2004) 105003; Interpretation of Quantum Field Theories with a Minimal Length Scale, *Phys. Rev. D* **73** (2006) 105013
- [23] S. Hossenfelder *et al*, Signatures in the Planck Regime, *Phys. Lett. B***575** (2003) 85-99
- [24] R. Akhoury and Y. P. Yao, Minimal Length Uncertainty Relation and the Hydrogen Spectrum, *Phys. Lett. B***572** (2003) 37-42
- [25] K. Nozari and T. Azizi, Some Aspects of Minimal Length Quantum Mechanics, *Gen. Rel. Grav.* **38** (2006) 735-742
- [26] K. Nozari and M. Karami, Minimal Length and Generalized Dirac Equation, *Mod. Phys. Lett. A* **20**(2005) 3095-3104
- [27] K. Nozari and S. H. Mehdipour, Gravitational Uncertainty and Black Holes Remnants, *Mod. Phys. Lett. A* **20** (2005) 2937-2948
- [28] Kourosh Nozari, Some Aspects of Planck Scale Quantum Optics, *Phys. Lett. B* **629** (2005) 41-52
- [29] M. Lubo, Quantum Minimal Length and Transplanckian Photons, *Phys. Rev. D***61** (2000) 124009; Minimal Length Uncertainty Principle and the Transplanckian Problem of Black Hole Physics, *Phys. Rev. D* **59** (1999) 044005
- [30] S. Benczik *et al*, Short Distance vs. Long Distance Physics: The Classical Limit of the Minimal Length Uncertainty Relation, *Phys. Rev. D* **66** (2002) 026003
- [31] L. N. Chang *et al*, The Effect of the Minimal Length Uncertainty Relation on the Density of States and the Cosmological Constant Problem, *Phys. Rev. D***65** (2002) 125028

- [32] G. Amelino-Camelia *et al*, The Search for Quantum Gravity Signals, arXiv:gr-qc/0501053 (AIP Conference Proceedings of the 2nd Mexican Meeting on Mathematical and Experimental Physics).
- [33] K. Nozari and S. H. Mehdipour, Wave Packets Propagation in Quantum Gravity, *Gen. Rel. Grav.***37** (2005) 1995
- [34] S. Kalyana Rama, Some Consequences of the Generalised Uncertainty Principle: Statistical Mechanical, Cosmological, and Varying Speed of Light, *Phys. Lett. B* **519** (2001) 103-110.
- [35] M. S. El Naschie, Statistical Mechanics of Multi-Dimensional Cantor Sets, Gödel Theorem and Quantum Spacetime, *Journal of the Franklin Institute* **330** (1993) 199-211
- [36] R. Gambini and J. Pullin, *Phys. Rev. D* **59** (1999)124021
- [37] J. Alfaro *et al*, *Phys. Rev. Lett.* **84** (2000) 2318
- [38] G. Amelino-Camelia *et al*, *Phys. Rev. D* **70** (2004) 107501
- [39] K. Nozari and B. Fazlpour, Thermodynamics of an Evaporating Schwarzschild Black Hole in Noncommutative Space, arxiv:hep-th/0605109
- [40] B. E. Schaefer, *Phys. Rev. Lett.* **82** (1999) 4964
- [41] <http://lhc-new-homepage.web.cern.ch/lhc-new-homepage/>
- [42] J. L. Feng and A. D. Shapere, *Phys. Rev. Lett.***88** (2002) 021303; A. Ringwald and H. Tu, *Phys. Lett. B***525**(2002)135; *Phys. Lett. B***529** (2002) 1; M. Ave, E. J. Ahn, M. Cavagli'a and A. V. Olinto, *Phys. Rev. D***68**(2003)043004; S. I. Dutta, M. H. Reno and I. Sarcevic, *Phys. Rev. D***66** (2002)033002; R. Emparan, M. Masip and R. Rattazzi, *Phys. Rev. D***65**(2002)064023; A. Mironov, A. Morozov and T. N. Tomaras, arXiv:hep-ph/0311318; P. Jain, S. Kar, S. Panda and J. P. Ralston, *Int. J. Mod. Phys. D***12**(2003)1593; M. Cavagli'a, *Int. J. Mod. Phys. A***18** (2003) 1843; G. Landsberg, arXiv:hep-ex/0310034; P. Kanti, arXiv:hep-ph/0402168.